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**10MAT41**

# Fourth Semester B.E. Degree Examination, Aug./Sept.2020
# Engineering Mathematics – IV

Time: 3 hrs. Max. Marks:100

**Note:** ***Answer any FIVE full questions, selecting at least TWO full questions from each part.***



## PART – A

1 a. Using Taylor's series method, solve the differential equation $\frac{dy}{dx} = x^2y + 1$ with $y(0) = 0$ at $x = 0.4$. Consider terms up to fourth degree. **(06 Marks)**

b. Solve the differential equation $\frac{dt}{dx} = -xy^2$ under the initial condition $y(0) = 2$, by using the modified Euler's method at $x = 0.1$ and $x = 0.2$. Take the step size $h = 0.1$ and carryout two modifications at each step. **(07 Marks)**

c. Apply Adams-Bashforth method to solve the equation $\frac{dy}{dx} = x^2(1+y)$ given $y(1) = 1$, $y(1.1) = 1.233$, $y(1.2) = 1.548$, $y(1.3) = 1.979$. Evaluate $y(1.4)$. **(07 Marks)**

2 a. Solve the differential equations:
$\frac{dy}{dx} = 1 + xz$ , $\frac{dz}{dx} = -xy$ for $x = 0.3$ using fourth order Runge-Kutta method. Initial values are $x = 0$, $y = 0$, $z = 1$, Take $h = 0.3$. **(06 Marks)**

b. Apply Picard's method upto third approximation to find y and z for the equation $\frac{d^2y}{dx^2} = 1 + \frac{dy}{dx}$ given $y(0) = 1 = y'(0)$. **(07 Marks)**

c. Apply Milne's method to compute $y(0.8)$ given that $\frac{d^2y}{dx^2} = 1 - 2y\frac{dy}{dx}$ and the following initial values $y(0) = 0$, $y(0.2) = 0.02$, $y(0.4) = 0.0795$ $y(0.6) = 0.1762$, $y'(0) = 0$, $y'(0.2) = 0.1996$, $y'(0.4) = 0.3937$, $y'(0.6) = 0.5689$. **(07 Marks)**

3 a. Derive Cauchy-Riemann equations in Cartesian form. **(06 Marks)**

b. Find the analytic function whose imaginary part is $e^x(x\sin y + y\cos y)$. **(07 Marks)**

c. If f(z) is an analytic function then prove that

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$$ **(07 Marks)**

4 a. Discuss the Transformation $W = e^z$. **(06 Marks)**

b. Find the bilinear transformation which maps the points 1, i, -1 onto the points i, 0, -1 respectively. **(07 Marks)**

c. Evaluate $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2(z-2)} dz$ where C is the circle $|z| = 3$, using Cauchy's integral formula. **(07 Marks)**

## PART – B

5 a. Find the solution of the Laplace's equation in cylindrical system leading to Bend's differential equation. **(06 Marks)**

b. Derive Rodrigue's formula

$$P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2 - 1)^n$$ **(07 Marks)**

c. Express the polynomial $2x^3 - x^2 - 3x + 2$ in terms of Legendre polynomials. **(07 Marks)**

6 a. A five figure number is formed by the digits 0, 1, 2, 3, 4 without repitition. Find the probability that the number is divisible by 4. **(06 Marks)**

b. If A and B are any two events with $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$, $P(A \cap B) = \frac{1}{4}$ find $P(A/B)$, $P(B/A)$, $P(A/\overline{B})$, $P(\overline{A}/\overline{B})$, $P(\overline{B}/\overline{A})$. **(07 Marks)**

c. The contents of three boxes are 1 white, 2 red, 3 green balls, 2 white, 1 red, 1 green balls and 4 white, 5 red, 3 green balls. Two balls are drawn from the box chosen at random. These are found to be one white and one green. Find the probability that the balls are from the third box. **(07 Marks)**

7 a. The probability distribution of a random variable X is given by the following table:

| X | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| P(X) | 0 | K | 2K | 2K | 3K | $K^2$ | $2K^2$ | $7K^2+K$ |

Find: $P(X < 5)$, $P(X \geq 6)$, $P(3 < X \leq 6)$. Also find mean and variance. **(06 Marks)**

b. Find mean and variance of Binomial distribution. **(07 Marks)**

c. In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find mean and standard deviation. Give A(0.5) = 0.19, A(1.4) = .42 where A(z) is the area under the standard normal curve from 0 to z. **(07 Marks)**

8 a. Explain the following terms:
i) Null hypothesis
ii) Significances level
iii) Confidence limits. **(06 Marks)**

b. A die was thrown 9000 times and a throw of 5 or 6 was obtained 3240 times on the assumption of random throwing, do the data indicate an unbiased die. **(07 Marks)**

c. A Machinist is making engine parts with axle diameter of 0.7inch. A random sample of 10 parts shows mean diameter 0.742 inch with a standard deviation of 0.04 inch. On the basis of this sample, would you say that the work is inferior? Given for $\rho = 9$, $T_{0.05} = 2.262$. **(07 Marks)**

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